

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 3987

Roll No.

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B.Tech.

(SEMESTER-IV) THEORY EXAMINATION, 2012-13

MATHEMATICS – III

Time : 3 Hours]

[Total Marks : 100

Note : Attempt questions from each section as indicated. The symbols have their usual meaning.

SECTION – A

1. All parts of this question are compulsory : 10 × 2 = 20
- (a) Find the constants a, b and c such that the function $f(z) = -x^2 + xy + y^2 + i(ax^2 + bxy + y^2)$ is analytic.
- (b) Evaluate the integral $\int_C \frac{e^{iz}}{z^3} dz$, where $C : |z| = 1$.
- (c) The first-four central moments of a distribution are 0, 2.5, 0.7 and 18.75. Comment on the kurtosis of the distribution.
- (d) The equations of two lines of regression are $3x + 12y = 19$ and $9x + 3y = 46$. Find the mean of x and the mean of y.
- (e) Enlist the methods by which Trend values can be determined.
- (f) Find the moment generating function of Poisson distribution.
- (g) Show that $hD \equiv -\sinh^{-1}(\mu\delta)$.



(h) Find the value of $\Delta^2(ab^{cx})$.

(i) Show that $y' = \frac{1}{h} \left[\Delta y - \frac{1}{2} \Delta^2 y + \frac{1}{3} \Delta^3 y - \frac{1}{4} \Delta^4 y + \dots \right]$

(j) Calculate the value of $\int_4^{5.2} \log_e x \, dx$ by Trapezoidal rule.

SECTION - B

2. Attempt any three parts :

3 × 10 = 30

(a) Using the method of contour integration, evaluate $\int_0^{\infty} \frac{dx}{(x^2 + a^2)^2}$.

(b) Find the multiple linear regression of x_1 on x_2 and x_3 from the data relating to three variables :

x_1	4	6	7	9	13	15
x_2	15	12	8	6	4	3
x_3	30	24	20	14	10	4

(c) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution.

(d) Perform four iterations of the Newton-Raphson method to obtain the approximate value of $(17)^{\frac{1}{3}}$ starting with initial approximation $x_0 = 2$.

(e) Find the value of $y(1.1)$, using Runge-kutta method of fourth order, given that $\frac{dy}{dx} = y^2 + xy$, $y(1) = 1.0$, take $h = 0.05$.

SECTION - C

Note : Attempt any two parts from each question.

5 × 10 = 50

3. (a) Using Cauchy's integral formula, evaluate

$$\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-3)} dz$$

where $C : |z| = 2$.

- (b) Prove that $\cosh\left(z + \frac{1}{z}\right) = a_0 + \sum_{n=1}^{\infty} a_n \left(z^n + \frac{1}{z^n}\right)$,

where $a_n = \frac{1}{2\pi} \int_0^{2\pi} \cos n\theta \cdot \cosh(2 \cos\theta) d\theta$.

- (c) State and prove Cauchy's Residue Theorem.

4. (a) Find the least squares fit of the form $y = a + bx^2$ to the following data :

x	-1	0	1	2
y	2	5	3	0

- (b) Show that the regression co-efficients are independent of the change of origin but not of scale.
- (c) Find the moment generating function for triangular distribution defined by

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2-x, & 1 \leq x \leq 2 \end{cases}$$

5. (a) If the variance of the Poisson distribution is 2, find the probabilities for $r = 1, 2, 3$ and 4 from the recurrence relation of the Poisson distribution. Also find $P(r \geq 4)$.

- (b) Given the following information in the usual notations :

$$n_1 = 7, n_2 = 6, S_1^2 = 6.21, S_2^2 = 5.23, \bar{x} = 30 \text{ and } \bar{y} = 28.$$

Test the hypothesis that the two samples have come from population having equal means.

- (c) 100 students of an engineering institute obtained the following grades in Mathematics paper :

Grade	A	B	C	D	E	Total
Frequency	15	17	30	22	16	100

Using χ^2 -test, examine the hypothesis that the distribution of grades is uniform.

6. (a) Find the missing term in the table :

x	2	3	4	5	6
$f(x)$	45.0	49.2	54.1	?	67.4

- (b) Show that the Regula-Falsi Method has linear rate of convergence.
- (c) Given the data $f(1) = 4$, $f(2) = 5$, $f(7) = 5$, $f(8) = 4$. Find the value of $f(6)$ and also the value of x for which $f(x)$ is maximum or minimum.
7. (a) Find the derivative of $f(x)$ at $x = 0.4$ from the following table :

x	0.1	0.2	0.3	0.4
$f(x)$	1.10517	1.22140	1.34986	1.49182

- (b) Use Picard's method to approximate the value of y when $x = 0.1$ given that $y = 1$ when $x = 0$ and $\frac{dy}{dx} = 3x + y^2$.
- (c) Solve the system : $x_1 + x_2 + x_3 = 1$,
- $$3x_1 + x_2 - 3x_3 = 5,$$
- $$x_1 - 2x_2 - 5x_3 = 10$$

by Crout's method.