(b) Derive the normal equations using the method of least squares to the curve of the form $y = ax + \frac{b}{x}$. Hence fit this curve to following data :

x	1	2	3	4
y	-1.5	0.99	3.88	7.66

(c) (i) If θ is the angle between the two regression lines, show that :

$$\tan\theta = \frac{1-r^2}{r} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}$$

Explain the significance when $r = 0$ and $r = \pm 1$.

5. Attempt any **two** parts of the following : **(10×2=20)**

(a) It is given that 2% of the electric bulbs manufactured by a company are defective. Using Poisson distribution, find the probability that a sample of 200 bulbs will contain (i) no defective bulb, (ii) two defective bulbs, (iii) at the most three defective bulbs.

(b) By using χ^2(Chi-Square) test, find out whether there is any association between income level and type of schooling :

Income	Public School	Govt. School
Low	200	400
High	1,000	400

(Given $\chi^2_{0.05} = 3.84$ for one degree of freedom).

(c) Discuss how control charts can be used in quality control of industrial products. The average percentage of defectives in 27 samples of size 1,500 each was found to be 13.7 %. Construct a suitable control chart for this problem.

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 3987 Roll No.

B. Tech.

(SEM. IV) THEORY EXAMINATION 2010-11

MATHEMATICS–III

Time : 3 Hours *Total Marks : 100*

Note : (1) Attempt **all** questions.

(2) Provide Chi-Square table.

1. Attempt any **four** parts of the following : **(5×4=20)**

(a) Determine an analytic function $f(z = u + iv$, in terms of z, whose real part is $e^{-x}(x \sin y - y \cos y)$.

(b) Prove that :

(i) $\oint_C \frac{dz}{z-a} = 2\pi i$

(ii) $\oint_C (z-a)^n dz = 0$, n is an integer $\neq -1$,

where C is the circle $|z - a| = r$.

(c) State Cauchy's integral formula. Hence evaluate :

$$\int_C \frac{\exp(i\pi z)}{(2z^2 - 5z + 2)} dz$$

where C is the unit circle with centre at origin and having positive orientation.

(d) Expand $f(z)=\frac{z}{(z^2-1)(z^2+4)}$ in Laurent series about in $1<|z|<2$.

(e) Determine the poles and residues at each poles of the function :

$$f(z)=\frac{z}{z^2-3z+2}$$

and hence evaluate

$$\oint_C f(z)\,dz \text{, where C is the circle } |z-2|=\frac{1}{2}.$$

(f) Apply calculus of residues to evaluate

$$\int_0^{2\pi}\frac{d\theta}{5+4\sin\theta}$$

2. Attempt any **four** parts of the following : **(5×4=20)**

(a) Solve $\cos x = 3x-1$ correct to three decimal places using the method of false position.

(b) Find the real root of the equation $\log_{10}x - x + 3 = 0$ correct to four decimal places using Newton-Raphson method.

(c) Using finite difference method, estimate the missing term in the table :

x	:	10	15	20	25	30	35
f(x)	:	43	–	29	32	–	77

(d) Prove the following relations :

(i) $\nabla\Delta = \delta^2 = \nabla\Delta$ (ii) $\nabla E = \Delta = \delta E^{\frac{1}{2}}$

where symbols have their usual meaning for finite difference.

(e) Find the Lagrange's interpolating polynomial f(x) for the following :

x	:	0	1	2	5
f(x)	:	2	3	12	147

(f) Estimate the population in 1895 from the following data :

Year	:	1891	1901	1911	1921	1931
Population (in thousands)	:	46	66	81	93	101

3. Attempt any **two** parts of the following : **(10×2=20)**

(a) Find the value of y(1.1) using Runge-Kutta method of order four for the differential equation :

$$\frac{dy}{dx}=y^2+xy,\ y(1)=1.0.\ \text{Take } h=0.05.$$

(b) Apply Crout's method to solve :

$$3x-y+2z=12$$
$$x+2y+3z=11$$
$$2x-2y-z=2.$$

(c) (i) Find f'(s) from the following table :

x	:	1	2	3	4	5
f(x)	:	10	26	50	82	122

(ii) The velocity 'v' of a particle at distance 's' from a point on its linear path is given in the following table :

s(m)	:	0	2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0
v(m/sec.)	:	16	19	21	22	20	17	13	11	9

Apply Simpson's rule to estimate the time taken by the particle to traverse the distance of 20 meters.

4. Attempt any **two** parts of the following : **(10×2=20)**

(a) The first four moments of a distribution about the value 5 of the variable are 2, 20, 40 and 50. Calculate the moments about the mean and comment upon the Skewness and Kurtosis of the distribution.