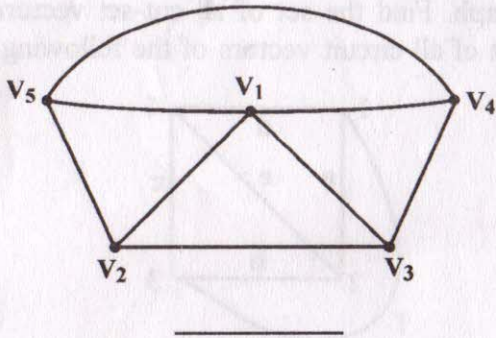


- (b) Define the adjacency matrix $X(G)$ of a graph. Let $X(G)$ be adjacency matrix of a simple graph G , then prove that ij th entry in X^r is the number of different edge sequences of r edges between vertices v_i and v_j .
- (c) If B is a circuit matrix of a connected graph G with n vertices and e edges, prove that $\text{rank } B = e - n + 1$.

5 Attempt any **two** parts of the following : **10×2=20**

- (a) Prove that in any directed graph the sum of the in-degrees of all the vertex equal to the sum of their out-degrees; and this sum is equal to the number of edges in the directed graph.
- (b) Prove that there n^{n-2} labeled trees with n vertices, $n \geq 2$.
- (c) Define the chromatic polynomial of graph G Find the chromatic polynomial of the following graph.



(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9972 Roll No.

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B. Tech.

(SEM. VI) EXAMINATION, 2008-09

GRAPH THEORY

Time : 3 Hours]

[Total Marks : 100

Note : Attempt all the questions.

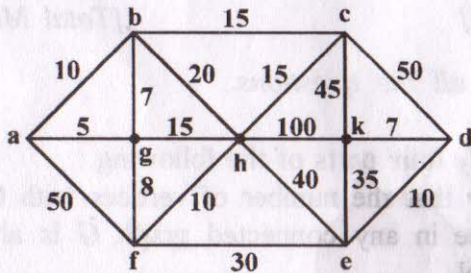
1 Attempt any **four** parts of the following : **5×4=20**

- (a) Show that the number of vertices with ODD degree in any connected graph G is always EVEN.
- (b) Define the basic operations : Union intersection and ring sum on suitably chosen examples of graphs.
- (c) Give the examples of two graphs G_1 and G_2 with 8 vertices. and edges ≥ 10 such that
(i) G_1 is both hamiltonian and eulerian
(ii) G_2 is neither hamiltonian nor eulerian.
- (d) Define isomorphism of graphs. Show that there are 11 non isomorphic graphs with 4 vertices.
- (e) Prove that if a graph has exactly two vertices of odd degree, there must be a path joining these two vertices.
- (f) If the intersection of two paths is a disconnected graph, show that the union of the two paths has atleast one circuit.



2 Attempt any **four** of the following : 5×4=20

- (a) Define radius, diameter and centre of a tree. Give an example of a tree for which the diameter is not equal to twice the radius.
- (b) Define a spanning tree for a connected graph. Find five spanning trees for K_5 .
- (c) Describe stepwise an algorithm for finding a minimum spanning tree in the following weighted graph



- (d) Find the minimum path between the vertices a and d (using Dijkstra algorithm) in the weighted graph of question 2(c).
- (e) Find total number of spanning trees for the Peterson's graph.
- (f) Define the rank and nullity of a graph. Find the rank and nullity of dodecahedron.

3 Attempt any **four** parts of the following : 5×4=20

- (a) Define the edge connectivity and the vertex connectivity of a graph. Construct a graph G with the following properties : edge connectivity of $G = 4$ Vertex connectivity of $G = 3$ and degree of every vertex of $G \geq 3$

- (b) Define the fundamental cut-sets of a graph G (w.r.t. a spanning tree) Find out all the fundamental cut sets of K_5 w.r.t any one of its spanning trees.
- (c) Define a non-separable graph G . Give an example of a non-separable graph with 8 vertices and 16 edges.
- (d) Define a planar graph. Establish the inequality for a planar graph G .
$$e \leq 3n - 6$$

where n is the number of vertices and e is the number of edges in G

- (e) If every region of a simple planar graph G (with n vertices and e edges) is bounded by k edges, show that
$$e = \frac{k(n-2)}{k-2}$$
- (f) Show that a complete graph with 4 vertices is self dual.

4 Attempt any **two** parts of the following : 10×2=20

- (a) Define a Cut-set vector and circuit vector of a graph. Find the set of all cut-set vectors and the set of all circuit vectors of the following graph.

