Printed Pages—3 **ECS505**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 2168 **Roll No.** | | | | | | | | | | |

B.Tech.

(SEM. V) ODD SEMESTER THEORY

EXAMINATION 2013-14

GRAPH THEORY

Time : 2 Hours *Total Marks : 50*

Note :– (1) Attempt **all** questions.

(2) Make suitable assumptions wherever necessary.

1. Attempt any **four** parts of the following : **(3×4=12)**

(a) When is a graph said to be regular ? Show that the number of vertices in a k-regular graph is even if k is odd.

(b) Find all nonisomorphic simple graphs of order 4.

(c) Define the following operations on the graphs with example :

(i) Product

(ii) Complement

(iii) Ring sum.

(d) In a park, jogging track is designed in such a way that there are four end points (say N, E, W, S). End point W is connected by two paths from end points N and S each and by single path from end point E. End points N and E are connected by single path. End points S and E are also connected by single path. Show that a jogging person can't return to its staring end point after walking through all the paths exactly once.

(e) Suppose G and G' are two graphs having n vertices. For what values of n is it possible for G to have more components and edges than G' ?

(f) Define the Hamiltonian Graph. Give two examples of Hamiltonian graph.

2. Attempt any **two** parts of the following : **(6×2=12)**

(a) Show that :

(i) A graph is a tree if and only if it is minimally connected.

(ii) A graph G with n vertices, n-1 edges and no circuits is connected.

(b) Define the radius and diameter of a graph. Show a tree in which its diameter is not equal to twice the radius. Under what condition does this inequality hold ? Elaborate.

(c) Write the Kruskal's algorithm for finding the minimum spanning tree of a graph, Illustrate the algorithm using an example.

3. Attempt any **two** parts of the following : **(6×2=12)**

(a) Define the edge-connectivity and vertex connectivity of a graph. Prove that the vertex connectivity of any graph G never exceed the edge connectivity of G.

(b) Show that the Kuratowski's second graph is nonplanar.

(c) (i) Determine the number of crossings and thickness of the Peterson graph.

(ii) Show that if G' is the geometric dual of a connected planar graph G, G is the geometric dual of G'.

4. Attempt any **four** parts of the following : **(3.5×4=14)**

(a) Prove that the set consisting of all the cut-sets and the edge-disjoint union of cut-sets (including the null set) in a graph G is an abelian group under the ring-sum operation.

(b) Define the chromatic polynomial of a graph. Find the chromatic polynomial of $K_{1,n}$.

(c) What is it meant by the basis Vectors of a graph ? Explain with an example.

(d) Show that every planar graph is 5-colorable.

(e) Define the incidence matrix of a connected graph with n vertices and e edges and prove that rank of incidence matrix of the graph is n – 1.

(f) Find the relationships among A_f, B_f, and C_f. Where A_f, B_f, and C_f represents incidence matrix, fundamental circuit matrix and fundamental cut set matrix of a connected graph, respectively.