(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 2165 Roll No. | | | | | | | | | | |

B. Tech.

(SEMESTER-V) THEORY EXAMINATION, 2012-13

DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 Hours] *[Total Marks : 100*

Note : Answer **all** the Sections.

Section – A

1. Attempt **all** question parts. **$10 \times 2 = 20$**

(A) Which of the following order of growth is correct ?

(a) $n^2 < n\log_2^n < n!$ (b) $n\log_2^n < n^3 < n!$

(c) $n < \log_2^n < n^2$ (d) $n < 2^n < n^3$

(B) The order of time for creating a heap of size n is

(a) O(n) (b) O(log n)

(c) O (n log n) (d) $O(n^2)$

(C) Quick sort exhibits its worst case behaviour when the input data is in ______ order.

(a) already sorted (b) reverse sorted

(c) random (d) do not have worst case

(D) Every internal node in a B-tree of minimum degree 2 can have ______ children.

(a) 2, 3 or 4 (b) 1, 2 or 3

(c) 2, 4 or 6 (d) 0, 2 or 4

(E) The second largest number from a set of n distinct numbers can be found in

(a) O(n) (b) O(1)

(c) $O(n^2)$ (d) O(log n)

(F) Back-Tracking and Branch-and-bound based solutions use ________.

(a) Spanning Tree (b) Decision Tree

(c) Binary Tree (d) State-space Tree

(G) A function t(n) is said to be in O(g(n)) if t(n)

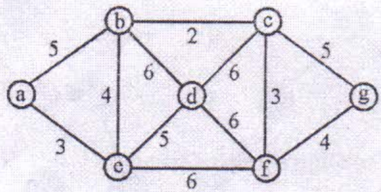
(a) is bounded both above and below by some constant multiples of g(n)

(b) is bounded above by some constant multiple of g(n) for all n

(c) is bounded below by some constant multiple of g(n) for all large n

(d) is bounded above by some function of g(n)

(H) Consider the following graph. Which of the following is NOT the sequence of edges added to the minimum spanning tree using Kruskal's algorithm ?



(a) (b, e), (e, f), (a, c), (b, c), (f, g), (c, d)

(b) (b, e), (e, f), (a, c), (f, g), (b, c), (c, d)

(c) (b, e), (a, c), (e, f), (b, c), (f, g), (c, d)

(d) (b, e), (e, f), (b, c), (a, c), (f, g), (c, d)

(I) A Hamiltonian circuit is

(a) the shortest cycle through all vertices of a graph.

(b) the fastest cycle through distinct vertices of a graph.

(c) a cycle that passes through all the vertices of a graph exactly once excepts the start node.

(d) cycle through points which form the smallest polygon that contains all points of a set of points.

(J) NP is the class of all decision problems whose randomly guessed solutions can be verified in

(a) Deterministic polynomial time

(b) Nondeterministic polynomial time

(c) NP hard time

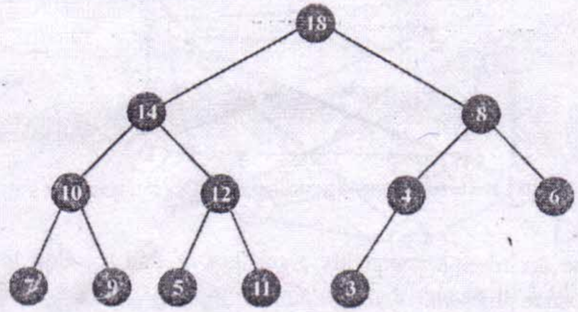
(d) NP complete time

Section – B

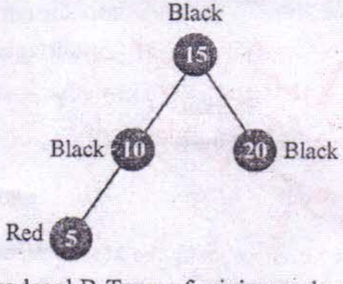
2. Attempt any **three** question parts. **10 × 3 = 30**

(a) (i) Describe the difference between **average-case** and **worst-case** analysis of algorithms, and give an example of an algorithm whose average-case running time is different from its worst-case running time. **(5)**

(ii) How will you represent a max-heap sequentially ? Explain with an example in the below given heap. **(5)**



(b) (i) Consider the following valid red-black tree, where "R" indicates a red node, and "B" indicates a black node. Note that the black dummy sentinel leaf nodes are not shown. Show the resulting red-black tree after inserting key 3 into and deleting 15 from the original tree. **(6)**



(ii) Show any two legal B-Trees of minimum degree 3 that represent {1, 2, 3, 4, 5, 6}. **(4)**

(c) (i) Suppose that undirected graph G = (V; E) has non-negative edge weights and these are raised by 1. Can the minimum spanning tree change ? Can shortest paths change ? Justify with proper example. (5)

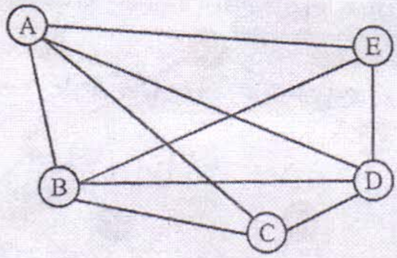
(ii) Show all the steps of Strassen's matrix multiplication algorithm to multiply the following matrices. (5)

$$X = \begin{bmatrix} 3 & 2 \\ 4 & 8 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 1 & 5 \\ 9 & 6 \end{bmatrix}$$

(d) (i) Consider the sum-of-subset problem, n = 4, Sum = 13, and $wt_1 = 3$, $wt_2 = 4$, $wt_3 = 5$ and $wt_4 = 6$. Find a solution to the problem using backtracking. Show the state-space tree leading to the solution. Also number the nodes in the tree in the order of recursion calls. (6)

(ii) State the implicit and explicit constraints of n-queens problem. (4)

(e) In the graph given below : (10)



(i) Write the triangle inequality algorithm to find solution for the Travelling Salesman problem.

(ii) Is the solution obtained from the algorithm optimal in all cases ?

(iii) For the graph given above, apply the algorithm starting from city A and obtain the solution. Properly indicate all the intermediate steps of execution of the algorithm.

Section – C

Attempt **all** questions : $10 \times 5 = 50$

3. Attempt any **two** parts : $(5 \times 2 = 10)$

(a) Solve the following recurrences using the Master method :

$T(1) = 0$

$T(n) = 9\,T(n/3) + n^3 \log n;\ n > 1$