

EC-II

(Following Paper ID and Roll No. to be filled in your Answer Book)

Paper ID : 2014073

Roll No.

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B. TECH.

Regular Theory Examination, (Odd Sem-III) 2016-17

MATHEMATICS - III

Time : 3 Hours

Max. Marks : 100

SECTION - A

1. Attempt all parts of this question. Each question carries two marks. (10×2=20)

- Evaluate $\int_{|z|=1/2} \frac{e^z}{z^2+1} dz$
- Find the residue of $f(z) = \cot z$ at its pole.
- Find the Z-transform of the sequence $\{a_n\}$.
- State the convolution theorem for inverse Z-transform.
- Discuss in brief the types of correlation.
- What do you understand by measures of Kurtosis, discuss in brief.

- g) Define order of convergence for finding out the root of an transcendental equation.
- h) For the data $[a, f(a)]$, $[a+h, f(a+h)]$ and $[a+2h, f(a+2h)]$, find $\Delta^2 f(a)$.
- i) Define a diagonal system of simultaneous linear algebraic equations.
- j) Write the formula for solving the differential equation $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$ by Runge-Kutta fourth order method.

SECTION - B

2. Attempt any three parts of the following:- (3×10=30)

- a) Use Calculus of Residue to evaluate the following integral

$$\int_{-\infty}^{\infty} \frac{\cos x}{(x^2 + a^2)(x^2 + b^2)} dx$$

- b) Find the Fourier transform of the following function defined for $a > 0$ by $f(t) = e^{-at^2}$

(2)

- c) Find the coefficient of correlation (r) and obtain the equation to the lines of regression for the following data:

| | | | | | |
|---|---|----|----|---|---|
| x | 6 | 2 | 10 | 4 | 8 |
| y | 9 | 11 | 5 | 8 | 7 |

- d) Using method of least squares, derive the normal equation to fit a parabola $y = a + bx + cx^2$ from the following data:

| | | | | | |
|---|----|----|----|----|----|
| x | 2 | 3 | 4 | 5 | 6 |
| y | 14 | 17 | 20 | 24 | 29 |

- e) Describe Picard's method for solving differential equation and hence solve the differential equation.

$$\frac{dy}{dx} = 1 + xy \text{ upto third approximation, when } y(0) = 0$$

SECTION - C

3. Attempt any two parts of the following : (2×5=10)

- a) Find the values of C_1 and C_2 such that the function $f(z) = x^2 + c_1 y^2 - 2xy + i(c_2 x^2 - y^2 + 2xy)$ is analytic. Also find $f(z)$.

- b) Find the poles (with its order) and residue at each poles of the following function:

$$f(z) = \frac{1-2z}{z(z-1)(z-2)^2}$$

- c) Find the Laurent series expansion of

$$f(z) = \frac{7z-2}{z(z+1)(z+2)} \text{ in the region } 1 < |z+1| < 3$$

4. Attempt any two parts of the following:- (2×5=10)

- a) Find the root of the equation $2x - \log_{10} x = 7$ which lies between 3.5 and 4.0, using method of false position (five iterations only).
- b) Using Newton's forward interpolation formula, find a polynomial function for $f(x)$ and hence evaluate $f(0.5)$, from the following data:

| | | | | | |
|--------|----|---|----|----|-----|
| x | 0 | 1 | 2 | 3 | 4 |
| $f(x)$ | -1 | 0 | 13 | 50 | 123 |

- c) Using Lagrange's method for interpolation, find $y(10)$ from the following data:

| | | | | |
|-----|----|----|----|----|
| x | 5 | 6 | 9 | 11 |
| y | 12 | 13 | 14 | 16 |

5. Attempt any two parts of the following:- (2×5=10)

- a) Evaluate the following integral, using Simpson's three - eight rule:

$$\int_0^6 \frac{dx}{1+x^2}$$

Taking 12 intervals.

- b) Apply Gauss-Seidal iteration method to solve the following equations (three iterations only)

$$20x + y - 2z = 17$$

$$3x + 20y - z = -18$$

$$2x - 3y + 20z = 25$$

- c) Find $f'(1.1)$ from the following data:

| | | | | | | |
|--------|-----|------|------|------|------|------|
| x | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |
| $f(x)$ | 0.0 | 0.12 | 0.55 | 1.29 | 2.43 | 4.00 |

6. Attempt any two parts of the following : (2×5=10)

- a) If for two random variables, x and y with same mean, the two regression lines are

NAS-301

$y = ax + b$ and $x = \alpha y + \beta$, then show that $\frac{b}{\beta} = \frac{1-a}{1-\alpha}$

Also find the common mean.

- b) The first four moments of a distribution about the value 4 of the variable are -1.5, 17, -30 and 108. Find the moments about the origin.
- c) Out of 800 families with 5 children each, how many families would be expected to have
- Three boys and two girls
 - At the most two girls.

Assume that probabilities for boys and girls are equal

7. Attempt any two parts of the following:- (2×5=10)

- a) Find the inverse Z-transform of

$$Z(z) = \frac{z}{z-1}, \quad |z| > 1$$

- b) Find the finite Fourier sine transform of

$$f(x) = x(\pi - x) \text{ in } 0 < x < \pi$$

- c) Using Z-transform, solve the following difference equation.

$$u_{n+2} + 2u_{n+1} + u_n = n \text{ with } u_0 = u_1 = 0$$

