Printed Pages : 7

NCS-302

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 110308

Roll No. | | | | | | | | | | |

B. Tech.

(SEM. III) (ODD SEM.) THEORY EXAMINATION, 2014-15

DISCRETE STRUCTURE AND GRAPH THEORY

Time : 3 Hours] [Total Marks : 100

1 Attempt any FOUR parts : 4×5=20

(a) Show that $R = \{(a,b) \mid a \equiv b \pmod{m}\}$ is an equivalence relation on Z. Show that if $x_1 \equiv y_1$ and $x_2 \equiv y_2$ then $(x_1 + x_2) \equiv (y_1 + y_2)$.

(b) Prove for any two sets A and B that, $(A \cup B)' = A' \cap B'$.

(c) Let R be binary relation on the set of all strings of 0's and 1's such that R={(a,b)| a and b are strings that have the same no. of 0's}. IS R is an equivalence relation? a Partial Ordering relation?

(d) If $f : A \to B$, $g : B \to C$ are invertible functions, then show that $gof : A \to C$ is invertible and $(gof)^{-1} = f^{-1}og^{-1}$.

(e) Prove by the principle of mathematical induction, that the sum of finite number of terms of a geometric progression,

$a+ar+ar^2+.....ar^{n-1} = a(r^n-1)/(r-1)$ if $r \neq 1$.

(f) Let $A=\{1,2,3,.......13\}$. Consider the equivalence relation on $A \times A$ defined by $(a,b)R(c,d)$ if $a+d = b+c$ Find equivalence classes of $(5,8)$.

2 Attempt any FOUR parts : 4×5=2

(a) Prove that $(Z_6, +_6)$ is an abelian group of order 6. where $Z_6 = \{0,1,2,3,4,5\}$.

(b) Let G be a group and let $a, b \in G$ be any elements. Then

(i) $(a^{-1})^{-1} = a$

(ii) $(a * b)^{-1} = b^{-1} * a^{-1}$.

(c) Prove that the intersection of two subgroups of a group is also subgroup.

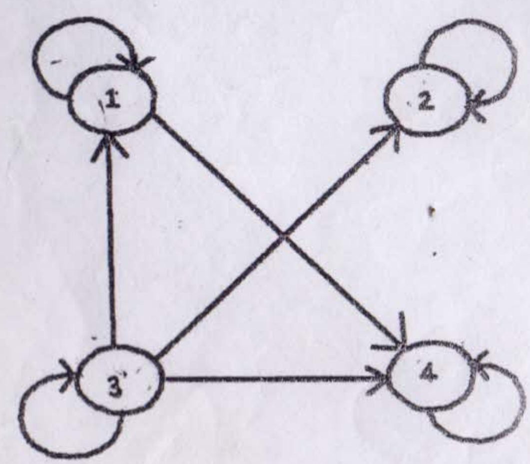
(d) Write and prove the Lagrange's theorem If a group $G = \{\ldots,-3,-2,-1,0,1,2,3,\ldots\}$ having the addition as binary operation. If H is a subgroup of group G where $x^2 \in H$ such that $x \in G$. What is H and its left Coset w.r.t. 1?

(e) Consider a ring$(R,+,\cdot)$ defined by $a \cdot a = a$, determine whether the ring is commutative or not.

(f) Show that every group of order 3 is cyclic?

3 Attempt any TWO parts : 2×10

(a) The directed graph G for a relation R on set $A=\{1,2,3,4\}$ is shown below:



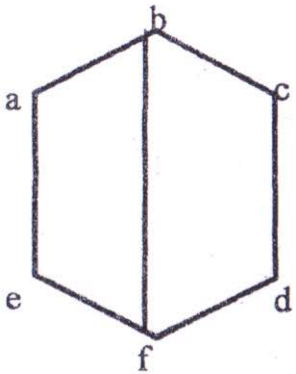
(i) Verify that (A,R) is a poset and find its Hasse diagram. poset → antisymmetric → reflexive

(ii) Is this a lattice? not

(iii) How many more edges are needed in the figure to extend (A,R) to a total order ? (1,2 & (2,4)

(iv) What are the maximal and minimal elements? (1,2) (3,4)

b) If the lattice is represented by the Hasse diagram given below :



(i) find all the complements of 'e'

(ii) Prove that the given lattice is bounded complemented lattice.

(c) (I) Consider the Boolean function

$f(x1,x2,x3,x4)=x1+(x2\cdot(x'1+x4)+x3\cdot(x'2+x'4))$

(i) simplify f algebraically.

(ii) Draw the logic circuit of the f and the reduction of the f.

(II) Write the expressions E1=(x+xy)+(x/y). and E2= x+ ((xy + y)/y), into

(i) prefix notation

(ii) postfix notation.

4 Attempt any TWO parts : 2×10=20

(a) (i) Show that $((p \vee q) \wedge \sim(\sim p \wedge (\sim q \vee \sim r))) \vee (\sim p \wedge \sim q) \vee (\sim p \vee r)$. Is a tautology without using truth table.

(ii) Rewrite the following arguments using quantifiers, variables and predicate symbols :

(a) All birds can fly.

(b) Some men are genius.

(c) Some numbers are not rational.

(d) There is a student who likes mathematics but not geography.

A

(b) "If the labour market is perfect then the wages of all persons in a particular employment will be equal. But it is always the case that wages for such persons are not equal therefore the labour market is not perfect". Test the validity of this argument using truth table.

B.

(c) Explain the following terms with suitable example :

(i) Conjunction $p \wedge q$

(ii) Disjunction $p \vee q$

(iii) Conditional $p \rightarrow q$

(iv) Converse $p \rightarrow q$ then $q \rightarrow p$

(v) Contrapositive. $p \rightarrow q$ $\sim q \rightarrow \sim p$

5 Attempt any TWO parts : **2×10=20**

(a) Solve the recurrence relation by the method of generating function

$$a_r - 7\,a_{r-1} + 10a_{r-2} = 0, \quad r \geq 2$$

Given $a_0 = 3$ and $a_1 = 3$

(b) Solve the recurrence relation

$$a_{r+2} - 5a_{r+1} + 6a_r = (r+1)^2$$

(c) Explain the following terms with example

(i) Homomorphism and Isomorphism graph.

(ii) Euler Graph and Hamiltonian graph.

(iii) Planar and Complete bipartite graph.

$A \rightarrow B$