(ii) Show that $\neg \forall x (P(x) \rightarrow Q(x))$ is logically equivalent to $\exists x (P(x) \wedge \neg Q(x))$, where all quantifiers have the same nonempty domain.

5. Attempt any **two** parts of the following : **(10×2=20)**

(a) Give the recursive definition of preorder, inorder, and postorder tree traversal. Also give an example of preorder, postorder and inorder traversal of a binary tree of your choice with at least 12 vertices.

(b) Solve $a_{n+2} - 5a_{n+1} + 6a_n = 2$ with initial condition $a_0 = 1$ and $a_1 = -1$.

(c) Write short notes on any **three** of the following :

(i) Bipartite Graphs

(ii) Generating function

(iii) Graph coloring

(iv) Pigeon hole principle.

Printed Pages—4 **ECS303**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 0111	**Roll No.**									

B.Tech.

(SEM. III) ODD SEMESTER THEORY EXAMINATION 2013-14

DISCRETE MATHEMATICAL STRUCTURES

Time : 3 Hours *Total Marks : 100*

Note :– (1) Attempt **all** questions.

(2) Make suitable assumptions wherever necessary.

1. Attempt any **four** parts of the following : **(5×4=20)**

(a) State whether the following sets are finite or infinite, countable or uncountable :

(i) $\{x \mid x \text{ is a positive integer}\}$.

(ii) $\{x \mid x \in \{a, b, ..., z\}\}$.

(iii) A set of lines passes through the origin.

(iv) $S = \{x \mid x^2 + 1 = 0\}$.

(b) How many different reflexive, symmetric relations are there on a set with three elements ?

(c) Let R denote a relation on the set of ordered pairs of positive integers such that (x, y) R (u, v) iff xv = yu. Show that R is an equivalence relation.

(d) Let $X = Y = Z = R$ and let the functions $f : X \to Y$ and $g : Y \to Z$ be defined as $f(x) = 2x + 3$ and $g(y) = y^2/3$. Show that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

(e) Use mathematical induction to show that if S is a finite set with n elements where n is a non-negative integer, then S has 2^n subsets.

(f) Explain the recursively defined functions with a suitable example.

2. Attempt any **two** parts of the following : **(10×2=20)**

(a) Let $W = \{1, \omega, \omega^2\}$ with the operation of ordinary multiplication on G be an algebraic structure, where ω is the cube root of unity such that $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$.

(i) Determine whether G is an Abelian.

(ii) Determine the order of each element in W.

(iii) Determine whether G is a cyclic group.

(b) Define the subgroup. If H is a subgroup of G such that $x^2 \in H$ for every $x \in G$, then prove that H is a normal subgroup of G.

(c) Define the permutation group and symmetric group. The dihedral group, D_3 is equal to S_3. Can you give a geometric explanation why ?

3. Attempt any **two** parts of the following : **(10×2=20)**

(a) Define a lattice. Give an example of a poset with five elements that is a lattice and an example of a poset with five elements that is not a lattice.

(b) State the commutative laws, associative laws and idempotent laws for lattices. Also prove these laws.

(c) Explain how K-maps can be used to simplify sum of products expansions in three Boolean variables. Use K-map to simplify the sum of products expansion.

$$x\,y\,z + x\,y'\,z + x\,y'\,z' + x'\,y\,z + x'\,y'\,z'.$$

4. Attempt any **two** parts of the following : **(10×2=20)**

(a) (i) Construct the truth table for $[(p \to q) \land (q \to r)] \to (p \to r)$.

(ii) Also show that above statement is a tautology by developing a series of logical equivalences.

(b) (i) Show that the hypothesis "If you send me an e-mail message, then I will finish writing the program," "If you do not send me an e-mail message, then I will go to sleep early," and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake feeling refreshed".

(ii) Show that the premises "A student in this class has read the book," and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book".

(c) (i) Show that $\forall x\,(P(x) \land Q(x))$ is logically equivalent to $\forall x\,P(x) \land \forall x\,Q(x)$, where all quantifiers have the same nonempty domain.