



(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 113606

Roll No.

--	--	--	--	--	--	--	--	--	--

B. Tech.

**(SEM. VI) THEORY EXAMINATION, 2014-15
GRAPH THEORY**

Time : 2 Hours]

[Total Marks : 50

Note : Attempt all questions.

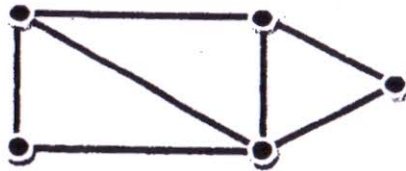
1 Attempt any four parts : 4×3=12

- (a) State a necessary and sufficient condition when a graph G is disconnected. Illustrate with an example.
- (b) State and verify :
 - (i) Which complete bipartite graphs are Hamiltonian?
 - (ii) Which complete graphs are Eulerian?
 - (iii) Is Peterson graph Hamiltonian?
- (c) Let T be a tree with 50 edges. The removal of certain edge from T yields two disjoint trees T_1 and T_2 . Given that the number of vertices in T_1 equals the number of edges in T_2 . Determine the number of vertices and number of edges in T_1 and T_2 .

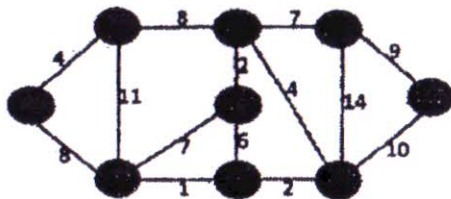
- (d) State and prove Handshaking Lemma.
 (e) State properties of cut-sets and discuss their applications.
 (f) Define the vector space associated with a graph.

2 Attempt any two parts : 2×6=12

- (a) For the given graph find out the vectors in the circuit subspace and cut-set subspace. Also find out the basis for each subspace.

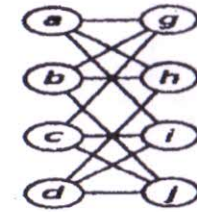
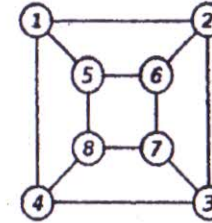


- (b) State and prove Euler's formula for planar graphs. Also show that in a simple connected planar graph with 6 vertices and 12 edges each of the regions is bounded by 3 edges.
 (c) Write the steps of Dijkstra's algorithm and use it to find the shortest path in the following graph from vertices 0 to 4.



3 Attempt any two parts : 2×6=12

- (a) Define incidence matrix of a graph with an example. Also prove that the rank of an incidence matrix of a graph with n vertices is $n-1$.
 (b) Define isomorphic graphs. Show that the following graphs are isomorphic.

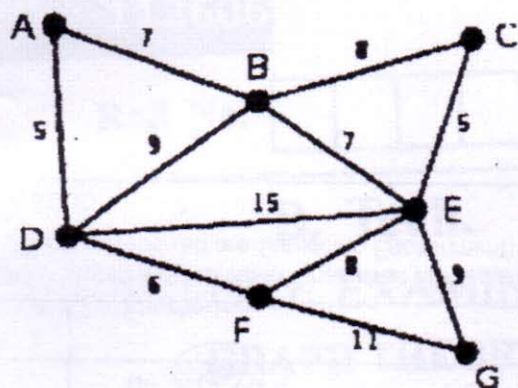


- (c) Let T be a graph with n vertices. Then prove that the following statements are equivalent :
 (i) T is a tree
 (ii) T contains no cycles and has $n-1$ edges
 (iii) T is connected and has $n-1$ edges
 (iv) T is connected and each edge is a bridge
 (v) Any two vertices of T are connected by exactly one path
 (vi) T contains no cycles, but the addition of any new edge creates exactly one cycle.

4 Attempt any four parts : 4×3.5=14

- (a) What do you mean by Geometrical dual of a graph? Prove that the complete graph with 4 vertices is self dual.

- (b) State and prove four color conjecture.
- (c) Using Kruskal's algorithm to find the minimal spanning tree of the following graph.



- (d) What are Kuratowski's Two Graphs ? Prove that these graphs are non-planar.
- (e) Find :
- The chromatic polynomial of $K_{2,m}$.
 - Three graphs with chromatic polynomial $\lambda^5 - 4\lambda^4 + 6\lambda^3 - 4\lambda^2 + \lambda$.
- (f) Prove that a binary tree with n vertices has $n-1$ edges.