

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 11050

Roll No.

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B.Tech.

(SEM. VI) THEORY EXAMINATION 2013-14

GRAPH THEORY

Time : 2 Hours

Total Marks : 50

Note :- Attempt all questions.

1. Answer any four parts : (4×3=12)
- (a) Prove that in a complete graph with n vertices there are $(n-1) / 2$ edge disjoint Hamiltonian circuit, if n is an odd number ≥ 3 .
 - (b) Prove that a simple graph with n vertices must be connected if it has more than $[(n-1)(n-2)]/2$ edges.
 - (c) Prove that in a every vertex of degree greater than one is a cut vertex.
 - (d) Prove that a non separable graph has a nullity $\mu = 1$ if and only if graph is a circuit.
 - (e) Prove that an Euler graph cannot have a cut set with an odd number of edges.
 - (f) Show that a Hamiltonian path is a tree.

2. Answer any **two** parts : (6×2=12)

- (a) Write Prim's algorithm to find minimal spanning tree.
- (b) Prove that a spanning tree T of a given weighted connected graph G is shortest spanning tree of G if and only if there exists no other spanning tree of G at a distance of one from T whose weight is smaller than that of T .
- (c) (i) Draw planar connected graph such that
- $e = 3n - 6$
 - $e < 3n - 6$
- (ii) Prove that in a nontrivial tree T there are at least two pendant vertices.

3. Answer any **three** parts of the following : (6×2=12)

- (a) Prove that a connected planar graph with n vertices and e edges has $n - e + 2$ regions.
- (b) If every region of a simple planar graph (with e edges and v vertices) embedded in a plane is bounded by k edges, show that $e = [k(n - 2)]/k - 2$
- (c) (i) Show that a complete graph of four vertices is self dual
- (ii) Using Kuratowski's theorem, show that Petersen's graph is nonplanar.

4. Answer any **four** parts of the following : (3.5×4=14)

- (a) Prove that covering h of a graph is minimal if and only if h contains no path of length three or more.
- (b) Prove that vertices of every planar graph can be properly colored with five colors.

(c) Show that the Chromatic polynomial of a graph of n vertices satisfies inequality

$$P_n(\lambda) \leq \lambda(\lambda-1)^{n-1}$$

- (d) If two graphs G_1 and G_2 are 1-isomorphic, prove that the rank of G_1 equals the rank of G_2 and nullity of G_1 equals the nullity of G_2 .
- (e) Show that a simple connected planar graph with 8 vertices and 13 edges cannot be bichromatic.
- (f) Prove that in a transport network G , the value of flow from source S to sink D is less than or equal to the capacity of any cut that separates S from D .