(b) Define the adjacency matrix X(G) of a graph. Let X(G) be adjacency matrix of a simple graph G, then prove that *ij* th entry in X^r is the number of different edge sequences of *r* edges between vertices v_i and v_j.

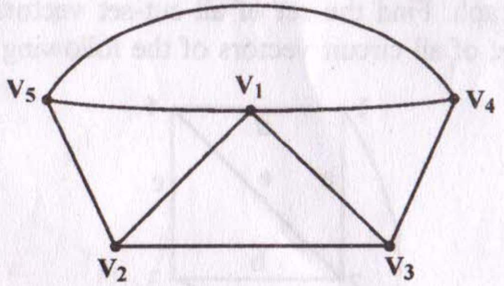
(c) If B is a circuit matrix of a connected graph G with *n* vertices and *e* edges, prove that rank $B = e - n + 1$.

5 Attempt any **two** parts of the following : 10×2=20

(a) Prove that in any directed graph the sum of the in-degrees of all the vertice equal to the sum of their out-degrees; and this sum is equal to the number of edges in the directed graph.

(b) Prove that there n^{n-2} labeled trees with *n* vartices, $n \geq 2$.

(c) Define the chromatic polynomial of graph G. Find the chromatic polynomial of the following graph.



Printed Pages : 4 **TMA – 011**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9972 Roll No.

B. Tech.

(SEM. VI) EXAMINATION, 2008-09

GRAPH THEORY

Time : 3 Hours] *[Total Marks : 100*

Note : Attempt ***all*** *the questions.*

1 Attempt any **four** parts of the following : 5×4=20

(a) Show that the number of vertices with ODD degree in any connected graph *G* is always EVEN.

(b) Define the basic operations : Union intersection and ring sum on suitably chosen examples of graphs.

(c) Give the examples of two graphs G_1 and G_2 with 8 vertices. and edges ≥ 10 such that

(i) G_1 is both hamiltonian and eulerian

(ii) G_2 is neigther hamiltonian nor eulerian.

(d) Define isomorphism of graphs. Show that there are 11 non isomorphic graphs with 4 vertices.

(e) Prove that if a graph has exactly two vertices of odd degree, there must be a path joining these two vertices.

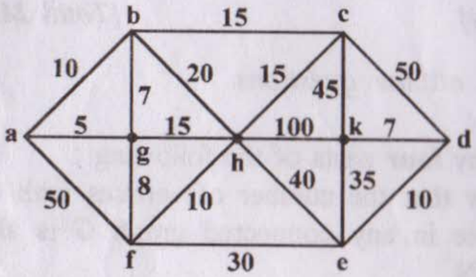
(f) If the intersection of two paths is a disconnected graph, show that the union of the two paths has atleast one circuit.

2 Attempt any **four** of the following : 5×4=20

(a) Define radius, diameter and centre of a tree. Give an example of a tree for which the diameter is not equal to twice the radius.

(b) Define a spanning tree for a connected graph. Find five spanning trees for K_5.

(c) Describe stepwise an algorithm for finding a minimum spanning tree in the following weighted graph



(d) Find the minimum path between the vertices *a* and *d* (using Dijkstra algorithm) in the weighted graph of question 2(c).

(e) Find total number of spanning trees for the Peterson's graph.

(f) Define the rank and nullity of a graph. Find the rank and nullity of dodeclahedron.

3 Attempt any **four** parts of the following : 5×4=20

(a) Define the edge connectivity and the vertex connectivity of a graph. Construct a graph *G* with the following properties : edge connectivity of *G* = 4 Vertex connectivity of *G* = 3 and degree of every vertex of $G \geq 3$

(b) Define the fundamental cut-sets of a graph *G* (w.r.t. a spanning tree) Find out all the fundamental cut sets of K_5 w.r.t any one of its spanning trees.

(c) Define a non-separable graph *G*. Give an example of a non-separable graph with 8 vertices and 16 edges.

(d) Define a planar graph. Establish the inequality for a planar graph *G*.

$$e \leq 3n - 6$$

where *n* is the number of vertices and *e* is the number of edges in *G*

(e) If every region of a simple planar graph *G* (with *n* vartices and *e* edges) is bounded by *k* edges, show that $e = \dfrac{k(n-2)}{k-2}$

(f) Show that a complete graph with 4 vertices is self dual.

4 Attempt any **two** parts of the following : 10×2=20

(a) Define a Cut-set vector and circuit vector of a graph. Find the set of all cut-set vectors and the set of all circuit vectors of the following graph.