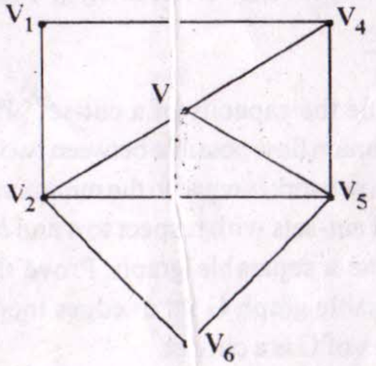
(c) Define reduced matrix A_f, fundamental circuit matrix b_f and fundamental cut-set matrix C_f of a connected graph G with n vertices and e edges. Derive the relationships among A_f, B_f and C_f.

5 Attempt any **two** of the following : 2×10=20

(a) Define the chromatic polynomial of a graph. Find the chromatic polynomial of the graph given below.



(b) State and prove five colour theorem.

(c) Define directed graph (digraph), simple digraph, asymmetric digraph, symmetric digraph, complete symmetric digraph and complete asymmetric digraph. Give example in each case. Also, prove that of the incidence matrix A(G) of digraph G, determinant of every square submatrix of A(G) is 1, −1 or 0.

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(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9972 Roll No. | | | | | | | | | | |

B. Tech.

(SEM. VI) EXAMINATION, 2007-08

GRAPH THEORY

Time : 3 Hours] *[Total Marks : 100*

Note : Attempt all the questions.

1 Attempt any **four** of the following : 5x4=20

(a) Define the degree of a vertex in a graph. Prove that the number of vertices of odd degree in a graph is always even.

(b) Prove that in a graph with n vertices and k components the max. number of edges cannot exceed $(n-k)(n-k+1)/2$.

(c) Define an eulerian and a hamiltonian graph. Give examples of eulerian nonhamilton graph G, and hamiltonian non-eulerian graph G_2 with No. of vertices ≥ 10.

(d) Define a connected graph. Prove that for a graph with exactly two vertices of odd degree, there must be a path joining these two vertices.

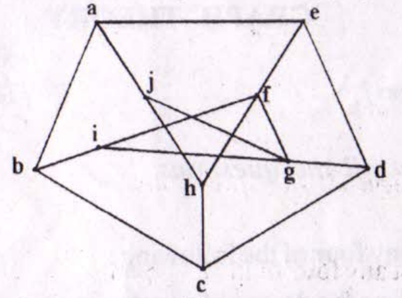
(e) Draw a graph G with a hamiltonian path but without a hamiltonian circuit with No. of vertices ≥ 20.

(f) Define a tree. Prove that a graph with n vertices, $n-1$ edges, and no circuits is connected.

2 Attempt any **four** of the following : **5×4=20**

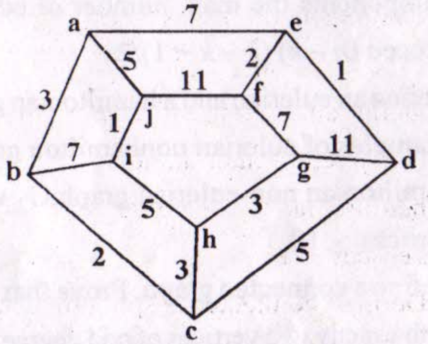
(a) Prove that every tree has one or two centres.

(b) Define a spanning tree of a graph. Find four spanning tress of the following. Peterson's graph.



(c) Prove that w.r.t. any of its spanning tress a connected graph with n vertices and e edges has $n-1$ tree branches and $e-n+1$ chords.

(d) Find a shortest spanning tree in a weighted-graph G, using the PRIM's algorithm where G is as follows :



(e) Construct a tree with 16 vertices, each corresponding to a spanning tree of a labelled completed graph with four vertices.

(f) Define fundamental circuit and cut sets. Find five fundamental circuits and fundamental cut sets of the graph in question 2(b).

3 Attempt any **four** of the following : **5×4=20**

(a) Define the vertex connectivity and edge connectivity of a graph. Prove that for a graph G with n vertices and e edges vertex connectivity $\leq$ edge connectivity $\leq \frac{2e}{n}$.

(b) Define the capacity of a cut-set . Prove that the maximum flow possible between two vertices a and b in a network is equal to the minimum of capacities of all cut-sets with respect to a and b.

(c) Define a separable graph. Prove that in a non-separable graph G set of edges incident on each vertex of G is a cut-set.

(d) Define a planar graph. Prove that a complete graph with five vertices in non planar.

(e) For a planar graph with n vertices and e edges, prove that $e \leq 3n-6$.

(f) Define thickness and crossing number of a graph. Find thickness and crossing numbers of the graphs K_5 and K_3, 3.

4 Attempt any **two** of the following : **2×10=20**

(a) Define a vector space of graph. Find five base and number of vectors in the vector space of graph of question 2(d). Also, find five cut-set vectors and five circuit vectors of this vector space.

(b) Define the adjacency matrix of a graph. Find the rank of the regular graph with n vertices and with degree $p(<n)$ of verty vertex.