(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 2168 Roll No. | | | | | | | | | | |

B.Tech.

(SEM. V) ODD SEMESTER THEORY EXAMINATION

2010-11

GRAPH THEORY

Time : 2 Hours *Total Marks : 50*

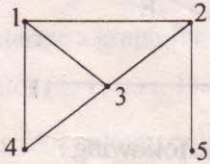
Note : (1) Attempt **all** questions.

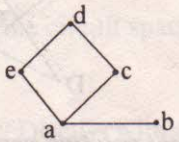
(2) Make suitable assumptions wherever necessary.

1. Attempt any **four** parts of the following : **(3×4=12)**

(a) What is a bipartite graph ? What is a complete bipartite graph ? Give an example of each.

(b) What do you understand by isomorphic graphs ? Whether the following graphs are isomorphic or not ?





(c) Define the following operations on the graphs with example :

(i) Union

(ii) Intersection

(iii) Ring sum

(d) Discuss the travelling salesman problem.

(e) Prove that a given connected graph G is an Euler graph if and only if all vertices of G are of even degrees.

(f) Give an example of a non Hamiltonian graph with 10 vertices such that for every pair of non adjacent vertices u and v,

$$d(u) + d(v) \geq 9$$

2. Attempt any **two** parts of the following : **(6×2=12)**

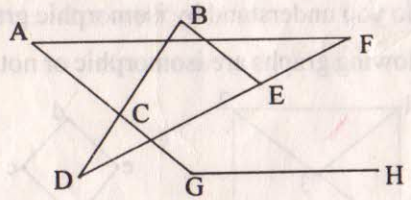
(a) Show that :

(i) If in graph G, there is one and only one path between every pair of vertices, G is tree.

(ii) A connected graph with n vertices and n-1 edges is a tree.

(b) Find all the fundamental circuits and cut sets of $K_{3,3}$ and K_5.

(c) Find at least three spanning trees of the following graph.



3. Attempt any **two** parts of the following : **(6×2=12)**

(a) Define the edge-connectivity and vertex connectivity of a graph. Prove that for a graph

vertex connectivity $\leq$ edge connectivity $\leq 2e/n$.

where e and n are number of edges and vertices in graph respectively.

(b) Describe an algorithm to detect the planarity of a graph. Detect planarity of K_5.

(c) Seven types of chemicals are to be shipped in five vans. There are three containers storing each type of chemical, and the capacities of the five vans are 6,4,5,4 and 3 containers respectively. For the security reasons, no van can carry more than one container of the same chemical. Determine whether it is possible to ship all 21 containers in the five vans.

4. Attempt any **four** parts of the following : **(3.5×4=14)**

(a) Prove that the set consisting of all the circuits and the edge-disjoint union of circuits (including the null set) in a graph G is an abelian group under the ring-sum operation.

(b) Define the chromatic number of a graph. Prove that every tree with two or more vertices is 2-chromatic.

(c) Define a circuit vector and a cut set vector of a connected graph. Prove that a circuit vector and a cut set vector are orthogonal to each other w.r.t. mod 2 arithmetic.

(d) Characterize a graph for which the circuit space contains the vector (1, 1, ..., 1).

(e) What is four colour conjecture ? Discuss with example.

(f) Explain the matching and covering of a graph.