

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9601

Roll No.

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B.Tech.

**(SEM. I) ODD SEMESTER THEORY EXAMINATION 2012-13
MATHEMATICS—I**

Time : 3 Hours

Total Marks : 100

SECTION—A

1. All parts for this question are compulsory : **(2×10=20)**

- (a) Find the 8th derivative of $x^2 e^x$.
- (b) If $x^2 = au + bv$, $y^2 = au - bv$, then find $\left(\frac{\partial u}{\partial x}\right)_y \cdot \left(\frac{\partial x}{\partial u}\right)_y$.

(c) Find the stationary points of
 $f(x, y) = 5x^2 + 10y^2 + 12xy - 4x - 6y + 1$.

(d) If $x = u(1 + v)$, $y = v(1 + u)$, then find the Jacobian of u, v with respect to x, y .

(e) Reduce the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ into normal form.

(f) Prove that the matrix $A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary.

(g) Evaluate $\int_0^1 \int_0^{x^2} x e^y dx dy$.

(h) Evaluate $\Gamma(-3/2)$.

(i) Find the value of m if $\vec{F} = mx\hat{i} - 5y\hat{j} + 2z\hat{k}$ is a solenoidal vector.

6. (a) Prove that : $\sqrt{\pi} \Gamma(2n) = 2^{2n-1} \Gamma(n) \Gamma\left(n + \frac{1}{2}\right)$,

where n is not a negative integer or zero.

(b) Change the order of integration and hence evaluate

$$\int_0^\infty \int_0^y y e^{-y^2/x} dx dy.$$

(c) Find the area of the region occupied by the curves $y^2 = x$ and $y^2 = 4 - x$.

7. (a) Show that the vector field $\vec{F} = yz\hat{i} + (zx + 1)\hat{j} + xy\hat{k}$ is conservative. Find its scalar potential. Also find the work done by \vec{F} in moving a particle from $(1, 0, 0)$ to $(2, 1, 4)$.

(b) Prove that :
 $\text{Curl}(\vec{F} \times \vec{G}) = \vec{F} \text{div } \vec{G} - \vec{G} \text{div } \vec{F} + (\vec{G} \cdot \nabla)\vec{F} - (\vec{F} \cdot \nabla)\vec{G}$.

(c) If $\text{div} [f(\vec{r})\vec{r}] = 0$, where \vec{r} is the position vector of a point (x, y, z) , then find $f(\vec{r})$.

- (j) Find the unit normal at the surface $z = x^2 + y^2$ at the point $(1, 2, 5)$.

SECTION—B

2. Attempt any **three** parts of the following : **(3×10=30)**

- (a) If $y = (x + \sqrt{1+x^2})^m$, then find the n^{th} derivative of y at $x = 0$.
- (b) Find the maximum and minimum distance of the point $(1, 2, -1)$ from the sphere $x^2 + y^2 + z^2 = 24$.
- (c) Find the eigen values and eigen vectors of the following matrix :

$$\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

- (d) Evaluate $\iiint_V (ax^2 + by^2 + cz^2) dx dy dz$ where V is the region bounded by $x^2 + y^2 + z^2 \leq 1$.
- (e) Verify Gauss's divergence theorem for the function $\vec{F} = x^2\hat{i} + z\hat{j} + yz\hat{k}$ over unit cube.

SECTION—C

Attempt any **two** parts from each question of this section. All questions are compulsory. **[(2×5)×5=50]**

3. (a) State and prove Euler's theorem for homogeneous functions.
- (b) Expand $f(x, y) = e^x \tan^{-1} y$ in powers of $(x-1)$ and $(y-1)$ upto two terms of degree 2.

- (c) If $z = f(x, y)$ where $x = e^u \cos v$, $y = e^u \sin v$, prove that

$$\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 = e^{-2u} \left[\left(\frac{\partial f}{\partial u}\right)^2 + \left(\frac{\partial f}{\partial v}\right)^2 \right]$$

4. (a) If $x + y + z = u$, $y + z = uv$, $z = uvw$, then find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$.
- (b) The two sides of a triangle are measured as 50 cm and 70 cm, and the angle between them is 30° . If there are possible errors of 0.5% in the measurement of the sides and 0.5 degree in that of the angle, find the maximum approximate percentage error in measuring the area of the triangle.
- (c) Show that $u = y + z$, $v = x + 2z^2$, $w = x - 4yz - 2y^2$ are not independent. Find the relation between them.
5. (a) Test the consistency and hence, solve the following set of equations :

$$10y + 3z = 0,$$

$$3x + 3y + 2z = 1,$$

$$2x - 3y - z = 5,$$

$$x + 2y = 4.$$

- (b) Using elementary transformations, find the rank of the following matrix :

$$A = \begin{bmatrix} -2 & -1 & 3 & -1 \\ 1 & 2 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

- (c) Examine the following vectors for linearly dependent and find the relation between them, if possible :

$$X_1 = (1, 1, -1, 1), X_2 = (1, -1, 2, -1), X_3 = (3, 1, 0, 1).$$