

- (b) Show that the following equations are consistent and solve them :

$$x + 2y - z = 3;$$

$$3x - y + 2z = 1;$$

$$2x - 2y + 3z = 2;$$

$$x - y + z = -1.$$

- (c) Find the eigen vectors for the matrix  $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ .

6. (a) Prove that :

$$\int_0^{\pi/2} \frac{dx}{\sqrt{\sin x}} \cdot \int_0^{\pi/2} \sqrt{\sin x} dx = \pi.$$

- (b) Evaluate  $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2 + y^2) dx dy$  by changing into

polar coordinates.

- (c) Determine the area bounded by the curves  $xy = 2$ ,  $4y = x^2$  and  $y = 4$ .

7. (a) Find the directional derivative of  $\text{div}(\text{grad } f)$  at the point  $(1, -2, 1)$  in the direction of the normal to the surface  $xy^2z = 3x + z^2$ , where  $f = 2x^3y^2z^4$ .

- (b) Find the work done in moving a particle in the force field :

$$\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$$

along the curve  $x^2 = 4y$  and  $3x^3 = 8z$  from  $x = 0$  to  $x = 2$ .

- (c) Find the constants  $a, b, c$  so that :

$$\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$$

is irrotational.

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9601

Roll No.

B. Tech.

(SEM. I) ODD SEMESTER THEORY

EXAMINATION 2013-14

MATHEMATICS—I

Time : 3 Hours

Total Marks : 100

SECTION—A

1. Attempt all parts of this question :

(10×2=20)

- (a) If  $z = u^2 + v^2$  and  $u = at^2$ ,  $v = 2at$ , then find  $\frac{dz}{dt}$ .

- (b) If  $y = x^2e^x$ , then find  $y_{nn}$ .

- (c) If  $u = lx + my$ ,  $v = mx - ly$ , then find  $\frac{\partial(x, y)}{\partial(u, v)}$ .

- (d) Find the stationary points of

$$f(x, y) = 5x^2 + 10y^2 + 12xy - 4x - 6y + 1.$$

- (e) Find the rank of the following matrix by reducing into Echelon form :

$$\begin{bmatrix} 3 & 1 & 2 \\ 6 & 2 & 4 \\ 3 & 1 & 2 \end{bmatrix}.$$

- (f) Find the eigen values of  $A^2$  where  $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 3 & 4 & 5 \end{bmatrix}$ .

(g) Evaluate  $\Gamma(3/4) \cdot \Gamma(1/4)$ .

(h) Use Dirichlet's integral to evaluate  $\iiint xyz \, dx \, dy \, dz$ , throughout the volume bounded by  $x=0, y=0, z=0$  and  $x+y+z=1$ .

(i) If  $\vec{r}$  is the position vector of a point, then find  $\text{Curl } \vec{r}$ .

(j) Find the unit normal to the surface  $x^2y + 2xz = 4$  at the point  $(2, -2, 3)$ .

### SECTION—B

2. Attempt any **three** parts of the following : **(3×10=30)**

(a) If  $y = e^{a \cos^{-1} x}$ , then find  $y_n$  for  $x = 0$ .

(b) A rectangular box, open at the top, is to have a volume of 32 c.c. Find the dimensions of the box requiring least material for its construction.

(c) Find the characteristic roots of the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}. \text{ Verify Cayley-Hamilton theorem for}$$

this matrix and hence find  $A^{-1}$ .

(d) Change the order of integration in the following integral and hence evaluate it :

$$\int_0^1 \int_{y^2}^{2-y} xy \, dx \, dy.$$

(e) Evaluate  $\iint_S (y^2 z^2 \hat{i} + z^2 x^2 \hat{j} + z^2 y^2 \hat{k}) \cdot \hat{n} \, dS$ , where S is

the part of the sphere  $x^2 + y^2 + z^2 = 1$  above the xy-plane and bounded by the xy-plane.

### SECTION—C

Attempt any **two** parts from each question of this section. All questions are compulsory : **[(2×5)×5=50]**

3. (a) Verify Euler's theorem for  $u = \frac{x(x^4 - y^4)}{x^4 + y^4}$ .

(b) Find the Taylor's series expansion of  $f(x, y) = x^2y + \sin y + e^x$  about the point  $(1, \pi)$  upto second degree terms.

(c) Trace the curve  $y = x(x^2 - 1)$ .

4. (a) If  $x = r \cos \theta, y = r \sin \theta$ , then prove that  $JJ' = 1$ .

(b) Find the maximum percentage error in the time period of a simple pendulum due to possible errors upto 1% in  $l$  and 2.5% in  $g$ .

(c) If  $u = y + z, v = x + 2z^2, w = x - 4yz - 2y^2$ , then find  $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ . Are  $u, v$  and  $w$  functionally related? If so,

find the relationship.

5. (a) Find the rank of the following matrix by reducing it into Normal form :

$$A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \end{bmatrix}.$$