

- (b) Reduce the following matrix to normal form and hence find its rank :

$$\begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$$

- (c) Show that the matrix

$$\begin{bmatrix} a + ic & -b + id \\ b + id & a - ic \end{bmatrix}$$

is unitary if and only if  $a^2 + b^2 + c^2 + d^2 = 1$ .

6. (a) Prove that

$$\int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{1}{4\sqrt{2}} \beta\left(\frac{1}{4}, \frac{1}{2}\right).$$

- (b) Evaluate

$$\iiint x^{\ell-1} y^{m-1} z^{n-1} dx dy dz,$$

where  $x > 0, y > 0, z > 0$  under the condition

$$\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \leq 1.$$

- (c) Find the area of one loop of the lemniscates

$$r^2 = a^2 \cos 2\theta.$$

7. (a) Find the directional derivative of  $\phi(x, y, z) = xy^2 + yz^3$  at the point  $(2, -1, 1)$  in the direction of the normal to the surface  $x \log z - y^2 + 4 = 0$  at  $(2, -1, 1)$ .

- (b) If all second order derivatives of  $\phi$  and  $\vec{v}$  are continuous, then show that

(i)  $\text{curl}(\text{grad } \phi) = \vec{0}$

(ii)  $\text{div}(\text{curl } \vec{v}) = 0$

- (c) Find the work done by the force

$$\vec{f} = (2y + 3)\hat{i} + xz\hat{j} + (yz - x)\hat{k}$$

when it moves a particle from the point  $(0, 0, 0)$  to the point  $(2, 1, 1)$  along the curve  $x = 2t^2, y = t$  and  $z = t^3$ .

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9601

Roll No.

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**B.Tech.**

(SEM. I) ODD SEMESTER THEORY EXAMINATION 2010-11  
**MATHEMATICS—I**

Time : 3 Hours

Total Marks : 100

**SECTION-A**

1. All parts of this question are compulsory :— **(2×10=20)**

- (a) If  $u = f\left(\frac{y}{x}\right)$  then

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \dots\dots\dots$$

- (b) The curve  $x^{2/3} + y^{2/3} = a^{2/3}$

is symmetrical about .....

**Indicate True or False of the following statements :**

- (c) (i) Two functions  $u$  and  $v$  are functionally dependent if their Jacobian with respect to  $x$  and  $y$  is zero.

(True/False)

- (ii) If  $f(x, y) = 1 - x^2y^2$ , then stationary point is  $(0, 0)$ .

(True/False)

- (d) (i) The minimum value of  $f(x, y) = x^2 + y^2$  is zero.

(True/False)

- (ii) If  $u, v$  are functions of  $r, s$  are themselves function of

$$x, y \text{ then } \frac{\partial(u, v)}{\partial(x, y)} = \frac{\partial(u, v)}{\partial(r, s)} \times \frac{\partial(x, y)}{\partial(r, s)}. \quad (\text{True/False})$$

**Pick the correct answer of the choices given below :**

- (e) The eigen values of  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$  are

- (a) 0, 0, 0    (b) 0, 0, 1    (c) 0, 0, 3    (d) 1, 1, 1

(f) The rank of the matrix  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$  is

- (a) 0      (b) 1      (c) 2      (d) 3

(g)  $\frac{\beta(m+1, n)}{\beta(m, n)}$  is equal to

- (a)  $\frac{m}{n}$       (b)  $\frac{m+1}{n}$       (c)  $\frac{m-1}{n}$       (d)  $\frac{m}{m+n}$

(h) The value of the integral  $\int_0^{\infty} e^{-x^2} dx$  is

- (a)  $\frac{2}{\sqrt{\pi}}$       (b)  $\frac{\sqrt{\pi}}{2}$       (c)  $\frac{\pi}{2}$       (d)  $\frac{2}{\pi}$

Fill up the blanks with the correct answer :

(i) The Gauss divergence theorem relates certain surface integrals to \_\_\_\_\_ (volume integrals/line integrals)

(j) The vector field  $\vec{F} = x\hat{i} - y\hat{j}$  is divergence free \_\_\_\_\_ (but not irrotational/and irrotational)

### SECTION-B

2. Attempt any **three** parts of the following : (10×3=30)

(a) If  $y = \sin(a \sin^{-1} x)$ . Find  $(y_n)_0$ .

(b) If  $u, v, w$  are the roots of the equation

$$(x-a)^3 + (x-b)^3 + (x-c)^3 = 0, \text{ then find } \frac{\partial(u, v, w)}{\partial(a, b, c)}$$

(c) Find the eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

(d) Change the order of integration in

$$I = \int_0^2 \int_{x^2/4}^{3-x} xy \, dy \, dx$$

and hence evaluate it.

(e) Find the volume enclosed between the two surfaces  $Z = 8 - x^2 - y^2$  and  $Z = x^2 + 3y^2$ .

### SECTION-C

Attempt any **two** parts from each question. All questions are compulsory. (5×2×5=50)

3. (a) Trace the curve  $y^2(a-x) = x^3$ .

(b) If  $Z = f(x+ct) + \phi(x-ct)$  show that  $\frac{\partial^2 Z}{\partial t^2} = c^2 \frac{\partial^2 Z}{\partial x^2}$ .

(c) Expand  $e^{ax} \sin y$  in the powers of  $x$  and  $y$  as far as terms of third degree.

4. (a) A rectangular box, open at the top, is to have a volume of 32 cubic feet. Determine the dimensions of the box requiring least material for its construction.

(b) If  $u_1 = \frac{x_2 x_3}{x_1}$ ,  $u_2 = \frac{x_3 x_1}{x_2}$  and  $u_3 = \frac{x_1 x_2}{x_3}$  find the value

$$\text{of } \frac{\partial(u_1, u_2, u_3)}{\partial(x_1, x_2, x_3)}$$

(c) Find the percentage of error in calculating the area of an

ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , when error of +1% is made in measuring the major and minor axes.

5. (a) Test for consistency and solve the following system of equations

$$2x - y + 3z = 8$$

$$-x + 2y + z = 4$$

$$3x + y - 4z = 0$$