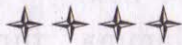


7. a) Verify Stokes theorem $\vec{F} = (2y+z, x-z, y-x)$ taken over the triangle ABC cut from the plane $x+y+z=1$ by the coordinate planes.

b) Verify Gauss Divergence theorem for $\int_c [(x^3 - yz)\hat{i} - 2x^2y\hat{j} + 2k] \hat{n} ds$ where S denotes the surface of cube bounded by the planes $x=0, x=a; y=0, y=a; z=0, z=a$.

c) If $\vec{A} = (xz^2\hat{i} + 2y\hat{j} - 3xz\hat{k})$ and $\vec{B} = (3xz\hat{i} + 2yz\hat{j} - z^2\hat{k})$
Find the value of $[\vec{A} \times (\nabla \times \vec{B})]$ & $[(\vec{A} \times \nabla) \times \vec{B}]$.



(Following Paper ID and Roll No. to be filled in your Answer Books)

Paper ID : 2012439

Roll No.

B.TECH.

Regular Theory Examination (Odd Sem - I), 2016-17

ENGINEERING MATHEMATICS-I

Time : 3 Hours

Max. Marks : 70

Note : The question paper contains three sections - A, B & C.
Read the instructions carefully in each section.

SECTION - A

Attempt all questions of this section. Each part carries 2 marks.

1. a) For what value of 'x', the eigen values of the given matrix A are real

$$A = \begin{bmatrix} 10 & 5+i & 4 \\ x & 20 & 2 \\ 4 & 2 & -10 \end{bmatrix} \quad (2)$$

b) For the given matrix $A = \begin{bmatrix} -5 & +3 \\ 2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
prove that $A^3 = 19A + 30I$. (2)

c) Find the maximum value of the function

$$f(xyz) = (z - 2x^2 - 2y^2) \text{ where } 3xy - z + 7 = 0. \quad (2)$$

d) If the volume of an object expressed in spherical coordinates as following :

$$V = \int_0^{2\pi} \int_0^{\pi} \int_0^1 r^2 \sin \phi \, dr \, d\phi \, d\theta \quad \text{Evaluate the value of } V. \quad (2)$$

e) Find the condition for the contour on $x - y$ plane where the partial derivative of $(x^2 + y^2)$ with respect to y is equal to the partial derivative of $(6y + 4x)$ with respect to x . (2)

f) The parabolic arc $y = \sqrt{x}$, $1 \leq x \leq 2$ is revolved around $x -$ axis. Find the volume of solid of revolution. (2)

g) For the scalar field $u = \frac{x^2}{2} + \frac{y^2}{3}$, Find the magnitude of gradient at the point $(1, 3)$. (2)

SECTION - B

Attempt any three parts of the following. Each part carries 7 marks.

2. a) i) Express $2A^5 - 3A^4 + A^2 - 4I$ as a linear

polynomial in A where $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$. (3)

ii) Reduce the matrix $P = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ to

diagonal form. (4)

b) i) If $u = \sin^{-1} \left(\frac{x^{1/4} + y^{1/4}}{x^{1/6} + y^{1/6}} \right)$ then evaluate the value

of $\left(x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} \right)$ (3)

ii) Trace the curve $x = a(\theta - \sin\theta)$,
 $y = a(1 - \cos\theta)$. (4)

c) i) Find the relation between u, v, w for the values
 $u = x + 2y + z; v = x - 2y + 3z;$
 $w = 2xy - zx + 4yz - 2z^2$. (3)

ii) Divide a number into three parts such that the
product of first, square of the second and cube
of third is maximum. (4)

d) i) Change the order of integration for
 $I = \int_0^1 \int_{x^2}^{2-x} xy \, dx \, dy$ and hence evaluate the same.
(3)

ii) Evaluate the triple integral
 $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (xyz) \, dx \, dy \, dz$. (4)

e) i) If $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$, then evaluate the
value of $\oint \vec{F} \cdot d\vec{r}$. (3)

ii) Find the directional derivative of $\left(\frac{1}{r^2}\right)$ in the
direction of \vec{r} where $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$. (4)

SECTION - C

Attempt all questions of this section, selecting any
two parts from each question. All questions carry
equal marks. (5×7=35)

3. a) If $I_n = \frac{d^n}{dx^n}(x^n \log x)$, show that $I_n = nI_{n-1} + \underline{n-1}$.

b) If $e^{-z/(x^2-y^2)} = x-y$ then show that
 $y \frac{\partial z}{\partial x} + x \frac{\partial z}{\partial y} = x^2 - y^2$.

c) If $w = \sqrt{x^2 + y^2 + z^2}$ & $x = \cos v$, $y = u \sin v$, $z = uv$,

then prove that $\left[u \frac{\partial w}{\partial u} - v \frac{\partial w}{\partial v} \right] = \frac{u}{\sqrt{1+v^2}}$.

4. a) If $x = v^2 + w^2$, $y = w^2 + u^2$, $z = u^2 + v^2$ then show

that $\frac{\partial(xyz)}{\partial(uvw)} \cdot \frac{\partial(uvw)}{\partial(xyz)} = 1$.

b) Express the function $f(xy) = x^2 + 3y^2 - 9x - 9y + 26$ as Taylor's Series expansion about the point (1, 2).

c) Find the percentage error in measuring the volume of a rectangular box when the error of 1% is made in measuring the each side.

5. a) If $A = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$ then evaluate the value of the expression $(A + 5I + 2A^{-1})$.

b) Find the eigen value of the matrix $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$

corresponding to the eigen vector $\begin{bmatrix} 101 \\ 101 \end{bmatrix}$.

c) Show that $A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w^2 & w \end{bmatrix}$ is a unitary matrix,

where w is complex cube root of unity.

6. a) Changing the order of integration in the double integral $I = \int_0^8 \int_{x/4}^2 f(xy) dy dx$ leads to the value

$I = \int_r^s \int_p^q f(xy) dx dy$. What is the value of q ?

b) Evaluate $\iiint x^2 yz dx dy dz$ through out the volume bonded by planes $x=0, y=0, z=0$ & $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

c) For the Gamma function, show that

$$\frac{\left(\frac{1}{3}\right) \left(\frac{5}{6}\right)}{\left(\frac{2}{3}\right)} = (2)^{1/3} \sqrt{\pi}$$