

(b) Using Green's theorem evaluate

$$\int_C (x^2 + xy)dx + (x^2 + y^2)dy$$

where C is the square formed by the lines  
 $y = \pm 1, x = \pm 1$

(c) Verify Stoke's theorem for

$\vec{F} = xy^2 \hat{i} + y \hat{j} + z^2 x \hat{k}$  for the surface of a  
rectangular lamina bounded by  
 $x = 0, y = 0, x = 1, y = 2, z = 0$



Printed Pages : 4

TAS104

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9916

Roll No.

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**B.Tech**

(SEM I) ODD SEMESTER THEORY EXAMINATION 2009-10

**MATHEMATICS I**

Time : 3 Hours]

[Total Marks : 100

**Note :** Attempt all questions.

1 Attempt any **two** parts of the following :

(a) Reduce the matrix :

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ -1 & 1 & -3 & -3 \\ 1 & 0 & 1 & 2 \\ 1 & -1 & 3 & 3 \end{bmatrix}$$

to column echelon form and find its rank.

(b) Verify the Cayley-Hamilton theorem for the matrix :

$$A = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 5 & 4 \\ -4 & 4 & 3 \end{bmatrix}$$

and hence find  $A^{-1}$ .



- (c) Find the eigenvalues and the corresponding eigen vectors of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

2 Attempt any **two** parts of the following :

- (a) If  $y = (x^2 - 1)^n$ , prove that

$$(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$$

- (b) If  $u = x^3 + y^3 + z^3 + 3xyz$ , show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 3u$$

- (c) If  $u = f(r)$  where  $r^2 = x^2 + y^2$ , prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$

3 Attempt any **two** parts of the following :

- (a) If  $x = r \sin \theta \cos \phi$ ,  $y = r \sin \theta \sin \phi$ ,  
 $z = r \cos \theta$ , show that

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin \theta$$

- (b) Determine the points where the function

$$f(x, y) = x^3 + y^3 - 3xy$$

has a maximum or minimum.

- (c) A rectangular box open at the top is to have a given capacity. Find the dimensions of the box requiring least material for its construction.

4 Attempt any **two** parts of the following :

- (a) Evaluate

$$\iint_A xy \, dx \, dy$$

where  $A$  is the domain bounded by  $x$ -axis, ordinate  $x = 2a$  and the curve  $x^2 = 4ay$ .

- (b) Find the volume bounded by the cylinder  $x^2 + y^2 = 4$  and the planes  $y + z = 4$  and  $z = 0$ .

- (c) Evaluate :

$$\iiint_R (x + y + z) \, dx \, dy \, dz, \text{ where}$$

$$R: 0 \leq x \leq 1, 1 \leq y \leq 2, 2 \leq z \leq 3$$

5 Attempt any **two** parts of the following :

- (a) Find a unit normal vector  $\hat{n}$  of the cone of revolution  $z^2 = 4(x^2 + y^2)$  at the point  $P: (1, 0, 2)$ .