5 Attempt any **TWO** parts : **10x2=20**

a) If $u=x+y+z$, $v=x^2+y^2+x^2$, $w=yz+zx+xy$. Prove that grad u, grad v and grad w are coplanar.

b) Verify Stokes theorem for $F=(x^2+y^2)I-2xyJ$ taken around the rectangle bounded by the lines $x=\pm a$, $y = 0, y = b$

c) Evaluate $\int_S (yzI+zxJ+xyK)$. ds where S is the surface of the sphere $x^2+y^2+x^2=a^2$ in the first octant.

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NAS-103

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 199125

Roll No.

B. Tech.

(SEM. I) (ODD SEM.) THEORY EXAMINATION, 2014-15

ENGG. MATHEMATICS-I

Time : **3** Hours] [Total Marks : **100**

1 Attempt any **FOUR** parts : **5x4=20**

a) If $y^{\frac{1}{m}}+y^{\frac{-1}{m}}=2x$ prove that

$$\left(x^2-1\right)y_{n-2}+\left(2n+1\right)xy_{n-1}+\left(n^2-m^2\right)y_n=0.$$

b) Prove that $xu_x+yu_y=\frac{5}{2}\tan u$ if

$$u=\sin^{-1}\left(\frac{x^3+y^3}{\sqrt{x}+\sqrt{y}}\right).$$

c) If $V=f(2x-3y,3y-4z,4z-2x)$ prove that

$$6V_x+4V_y+3V_z=0.$$

d) Find $\frac{du}{dt}$ as a total derivative and verify the result by direct substitution if $u = x^2 + y^2 + z^2$ and $x = e^{2t}$, $y = e^{2t}\cos 3t$, $z = e^{2t}\sin 3t$.

e) Trace the curve $y^2(2a - x) = x^3$.

f) Find the curve $r^2 = a^2 \cos 2\theta$.

2 Attempt any **TWO** parts : **10x2=20**

a) Expand $e^x \log(1+y)$ in powers of x and y upto terms of third degree.

b) A rectangle box open at the top is to have 32cubic ft. Find the dimensions of the box requiring least material for its construction.

c) Find $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$ if $x = \sqrt{vw}, y = \sqrt{uw}, z = \sqrt{uv}$ and $u = r\sin\theta\cos\phi, v = r\sin\theta\sin\phi, w = r\cos\theta$.

3 Attempt any **TWO** parts : **10x2=20**

a) Reduce A to Echelon form and then to its row canonical form where $A = \begin{pmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{pmatrix}$. Hence find the rank of A.

b) Verify Cayley-Hamilton theorem for $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$. Hence find A^{-1}.

c) Solve by calculating the inverse by elementary row operations : $x_1 + x_2 + x_3 + x_4 = 0$, $x_1 + x_2 + x_3 - x_4 = 4$, $x_1 + x_2 - x_3 + x_4 = -4$, $x_1 - x_2 + x_3 + x_4 = 2$.

4 Attempt any **TWO** parts : **10x2=20**

a) Determine the area bounded by the curves xy = 2, $4y=x^2$ and y = 4.

b) Change the order of integration and evaluate $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$.

c) Find the volume and the mass contained in the solid region in the first octant of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ if the density at any point $\rho(x,y,z) = kxyz$.